MATHEMATICS

TRANSFORMATIONS

K12 SCHOOLS
YOUR EDUCATION PORTAL

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TRANSLATIONS

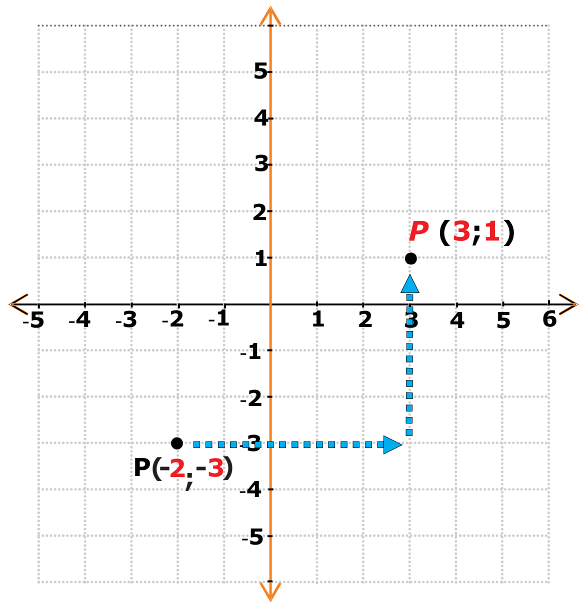
A translation refers to a horizontal and/or vertical change by a rule. The image is congruent to the original figure. The rule (x;y) → (x+a;y+b), covers both a horizontal and or vertical change.

Example 1

1.1 **Consider the following translations:**

1.1.1 ***P*(-2;-3) to *P′*(3;1)** has been translated according to the rule

(x;y) → (x+5;y+4) where P has moved 5 units horizontally to the right and 4 units vertically upwards.



Translating a figure does not change the size of the figure. The image is congruent to the original figure.

Example 2

2.1 **The image of P(2;3) , after the following translations.**

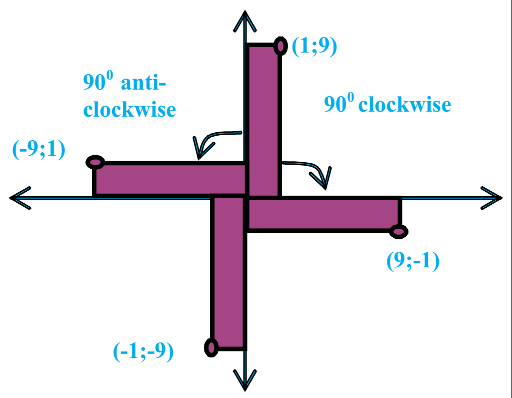
2.1.1 (x;y) → (x-3;y+1) will become *P′*(2-3;3+1) = *P′*(-1;4)

2.1.2 (x;y) → (x+6;y-4) will become *P′*(2+6;3-4) = *P′*(8;-1)

Example 3

Rotations of ± 90° and ±180°

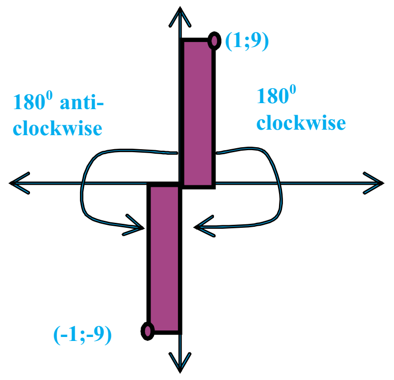
Consider the point (1;9) shown in the sketch.



3.1 The image of (1;9) rotated 90° anti-clockwise will become (-9;1).
Generally:
90° anti- clockwise or 90° becomes (x;y) → (-y;x)

3.2 The image of (1;9) rotated 90° clockwise will become (9;-1).
Generally:
90° clockwise or - 90° becomes (x;y) → (y;-x)

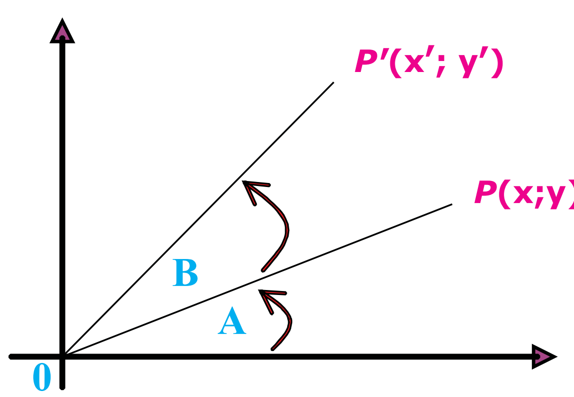
3.3 The image of (1;9) rotated 180° anti-clockwise or clock-wise will become (-1;-9).
Generally:
180° clockwise or anti-clockwise (x;y) → (-x;-y)



3.4 360° clockwise or anti-clockwise rotation through the origin leaves the figure or point unchanged.

A FORMULA FOR FINDING THE CO-ORDINATES OF A POINT AFTER ROTATION IN AN ANTI-CLOCKWISE DIRECTION

Consider OP = r = OP′ where $\hat{B}$ is the angle of rotation of OP through the origin.



By definition cos $= \frac{x}{r}$ and sin $= \frac{y}{r}$ giving x= rcos and y=rsin

cos(A + B)$=\frac{x'}{r}$ and sin(A + B)$= \frac{y'}{r}$

x′ =r.cos(A + B)
=r[cosAcosB - sinAsinB]
=(rcosA)cosB - (rsinA)sinB
=x.cosB-y.sinB since rcosA=x and rsinA=y

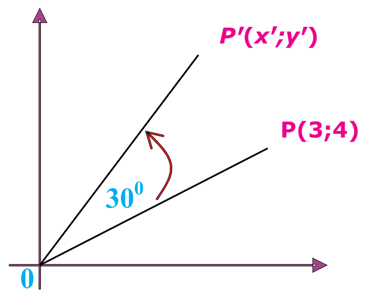
y′=r.sin(A + B)
=r[sinAcosB + cosAsinB]
=(rsinA)cosB + (rcosA)sinB
=y.cosB+x.sinB since rsinA=y and rcosA=x

The co-ordinates of P(x;y) after rotation about the origin is:
***P′*(xcosB - ysinB ; ycosB + xsinB)**

Example 4

Consider P(3;4) as shown.

P is rotated 30° anti-clockwise through the origin. The co-ordinates of point (3;4) after rotation about the origin through 30° is obtained as follows:



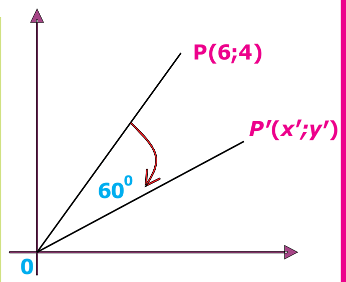
P′(xcosB - ysinB ; ycosB + xsinB)
=(3cos30° - 4sin30° ;4cos30° + 3sin30°)
$=\left\{3.\frac{\sqrt{3}}{2} - 4.\frac{\sqrt{3}}{2} + 3.\frac{1}{2}\right\}$
$=\left\{\frac{3\sqrt{3}}{2} - 2 ; 2\sqrt{3} + \frac{3}{2}\right\}$

Example 5

Consider P(6;4) as shown.

P is rotated 60° clockwise through the origin. The formula holds true for clockwise rotation through the origin, however remember the angle is now negative.

P′(xcosB - ysinB ; ycosB + xsinB)
=[6cos(-60°)-4sin(-60°);4cos(-60°)+6sin(-60°)]
=[6cos(60°) + 4sin(60°);4cos(60°) - 6sin(60°)]
$=\left\{6.\frac{1}{2} + 4.\frac{\sqrt{3}}{2} ; 4.\frac{1}{2} - 6.\frac{\sqrt{3}}{2}\right\}$
$=\left\{3 + 2\sqrt{3} ; 2 - 3\sqrt{3}\right\}$

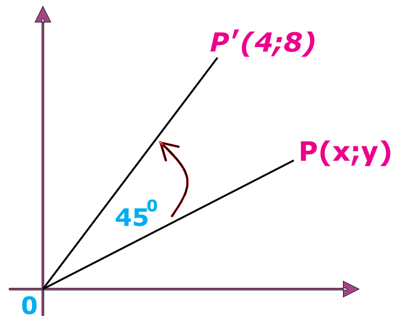


Alternatively you could use a 300° anti-clockwise rotation to obtain the same answer.

Example 6

In this example , as shown in the figure *P′*(4 ;8) results from P(x;y) being rotated 45° in an anti-clockwise direction through the origin.

To find P we proceed as follows:
The angle of rotation is reversed i.e we use -45° or if you prefer use 315° to get to the original position of P before rotation.

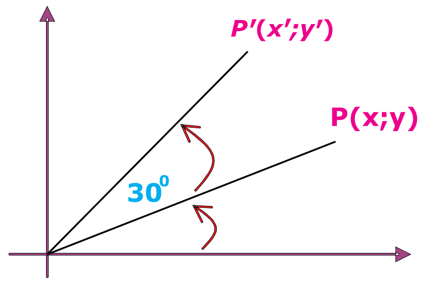


P(xcosB - ysinB ; ycosB + xsinB)
=[4cos(-45°)- 8sin(-45°);4cos(-45°)+8sin(-45°)]
=[4cos45° + 8sin45°; 4cos45°- 8sin45°]
$=\left\{4.\frac{\sqrt{2}}{2} + 8.\frac{\sqrt{2}}{2} ; 4.\frac{\sqrt{2}}{2} - 8.\frac{\sqrt{2}}{2}\right\}$
$=\left\{2\sqrt{2} + 4\sqrt{2} ; 2\sqrt{2} - 4\sqrt{2}\right\}$
$=\left\{6\sqrt{2} ; -2\sqrt{2}\right\}$

Example 7

Refer to the diagram alongside.

7.1 The co-ordinates of ,the image of P(x;y) rotated about the origin through an angle of 30° in the anti-clockwise direction , is found as follows:



P′(xcosB - ysinB ; ycosB + xsinB)
=x cos30° - y sin30°); y cos30° + x sin30°
$=\left\{x.\frac{\sqrt{3}}{2} - y.\frac{1}{2} ; y.\frac{\sqrt{3}}{2} + x.\frac{1}{2}\right\}$
$=\left\{\frac{\sqrt{3}}{2}x - \frac{1}{2}y ; \frac{\sqrt{3}}{2}y + \frac{1}{2}x\right\}$
$=\left\{\frac{\sqrt{3}}{2}x - \frac{y}{2} ; \frac{\sqrt{3}}{2}y + \frac{x}{2}\right\}$

7.2 *K′* is the image of K(4;3) under a rotation of 30° ,in the anti-clockwise direction, about the origin. We can now use the above result to find the co-ordinates of *K′*.

K′ $\left\{\frac{\sqrt{3}}{2}x - \frac{y}{2} ; \frac{\sqrt{3}}{2}y + \frac{x}{2}\right\}$
$=\left\{\frac{\sqrt{3}}{2}.4 - \frac{3}{2} ; \frac{\sqrt{3}}{2}.3 + \frac{4}{2}\right\}$
$=\left\{2\sqrt{3} - \frac{3}{2} ; \frac{3\sqrt{3}}{2} + 2\right\}$

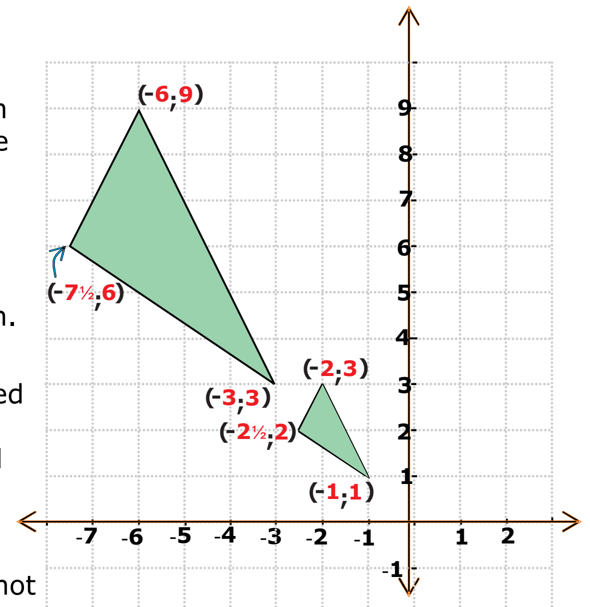
Example 8

In the diagram ΔABC has coordinates A(-2;3), and C(-1;1).

To enlargen ΔABC through the origin by a factor 3 we Proceed as follows:

Geometrically , from the origin to each vertex produce a line that will be 3 times the original length.

Another approach would be to multiply each ordered pair by the factor yielding *A′*(-6;9), *B′*(-7½ ;6) and *C′*(-3;3)



Note that the shape is preserved but the size is not preserved.

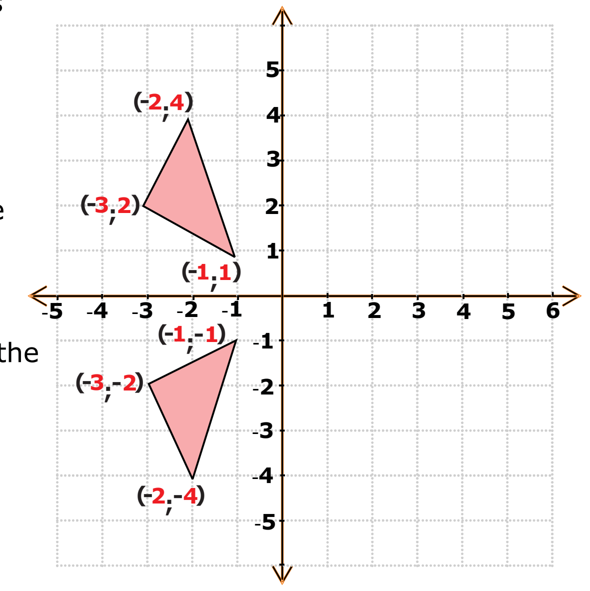
Useful Fact: When we enlarge by a factor k the area increases by a factor k².

Example 9

In the diagram ΔABC has co-ordinates A(-1;1), B(-3;2) and C(-2;4).

To reflect ΔABC about the x axis, we plot the reflected points that are equal in length to the x axis as shown in the diagram.

Generally (x;y) becomes (x;-y) upon reflection in the x axis.

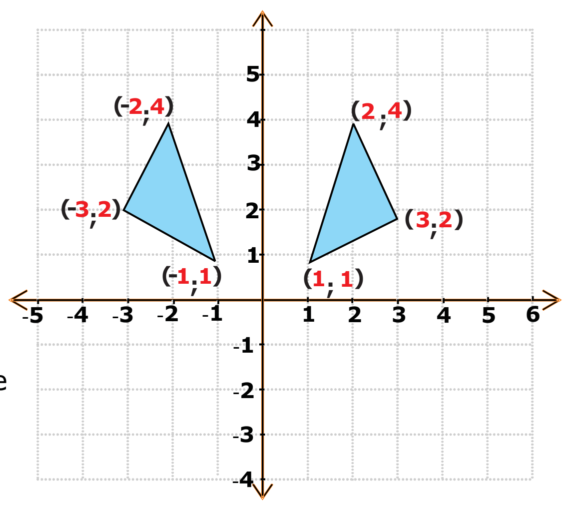


Example 10

In the diagram ΔABC has co-ordinates A(-1;1), B(-3;2) and C(-2;4).

To reflect ΔABC about the y axis, we plot the reflected points that are equal in length to the y axis as shown in the diagram.

Generally (x;y) becomes (-x;y) upon reflection in the y axis.

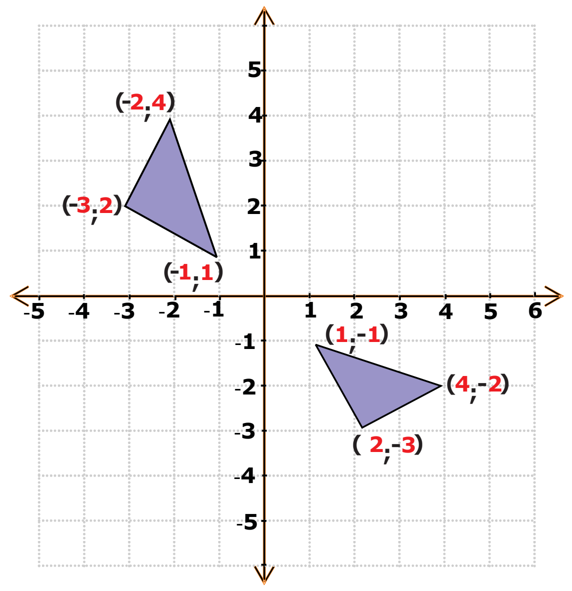


Example 11

In the diagram ΔABC has co-ordinates A(-1;1), B(-3;2) and C(-2;4).

To reflect ΔABC about the line y=x, we replace x with y and y with x.

Generally (x;y) becomes (y;x) upon reflection in the line y=x.



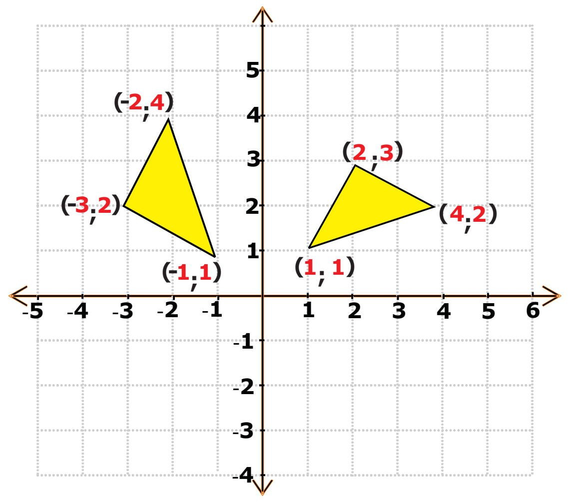
Example 12

In the diagram ΔABC has co-ordinates A(-1;1), B(-3;2) and C(-2;4).

In this example we will rotate ΔABC 90° clockwise about the origin.

(x;y) becomes (y;-x) upon a 90° clockwise rotation through the origin.

The co-ordinates will be:
***A*(-1;1) → *A′*(1;1)**
***B*(-3;2) → *B′*(2;3)**
***C*(-2;4) → *C′*(4;2)**



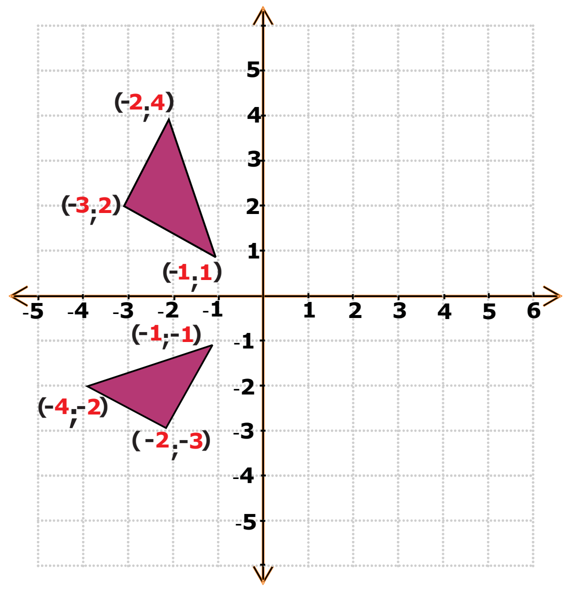
Example 13

In the diagram ΔABC has co-ordinates A(-1;1), B(-3;2) and C(-2;4).

In this example we will rotate ΔABC 90° anti-clockwise about the origin.

(x;y) becomes (-y;x) upon a 90° anti-clockwise rotation through the origin.

The co-ordinates will be:
***A*(-1;1) → *A′*(-1;-1)**
***B*(-3;2) → *B′*(-2;-3)**
***C*(-2;4) → *C′*(-4;-2)**



Example 14

In the diagram ΔABC has co-ordinates A(-1;1), B(-3;2) and C(-2;4).

In this example we will rotate ΔABC 60° anti-clockwise through the origin.

Using our formula below we obtain the co-ordinates:
***A*(-1;1) → *A′*(-1,4;-0,4)**
***B*(-3;2) → *B′*(-3,2;-1,6)**
***C*(-2;4) → *C′*(-4,5;0,3)**