K12 SCHOOLS
YOUR EDUCATION PORTAL

MATHEMATICS
CALCULUS

Co-ordinator: **M. KHAN**
Southlands Secondary

Assessment Standards

You must be able to:

1. establish the derivatives of functions from first principles.
2. generalize to derivative of $f(x) = x^n$
3. use the following rules:

a. $\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$

b. $\frac{d}{dx}[k.f(x)] = k\frac{d}{dx}[f(x)]$

What is this section about?

Calculus

Finding Derivatives

Equations of Tangents

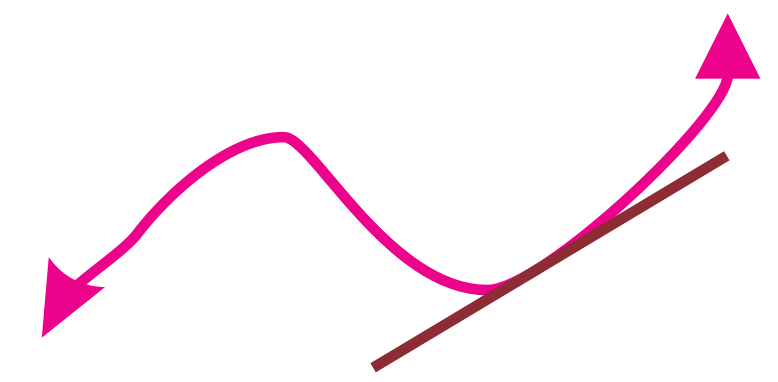
Curve Sketching

Maxima Minima

Rates of Change

Limits

A. The Fundamental Question of Calculus



What is the slope of the tangent line to the function???

The derivative is the slope of the tangent to the curve

SECTION: Finding Derivatives using first principles

THEORY: **Use formula in 3 phases.**

Step 2 **Step 1**

$$f^{1}(x) = \lim_{h \Rightarrow 0} \frac{f(x+h) \qquad f(x)}{h}$$

Step 1: **Given the function**
Step 2: **Replace (x) with (x + h) in the function.**
Step 3: **Find the difference and apply the formula.**

The following examples must be practised until they are mastered! This is not negotiable for the Final Examinations.

Example 1: Find the derivative of f(x) = 5 from first principles

$f(x) = 5$
$f(x + h) = 5$
$f(x = h) - \quad f(x) = 0$

$$f^{1}(x) = \lim_{h \Rightarrow 0} \frac{f(x+h) \qquad f(x)}{h}$$
$$= \lim_{h \Rightarrow 0} \frac{0}{h}$$
$$= 0.$$

The above result clearly agrees with our knowledge that the gradient of a constant function is 0.

Example 2: Determine $f^{1}(x)$ if $f(x) = x$ by using the definition

$f(x) = x$
$f(x + h) = x = h$
$f(x = h) - \quad f(x) = h$

$$f^{1}(x) = \operatorname{Lim}_{h \Rightarrow 0} \frac{f(x+h) \qquad f(x)}{h}$$
$$= \lim_{h \Rightarrow 0} \frac{h}{h}$$
$$= 1$$

The gradient of a linear function (y = mx + c) is the m value from past knowledge

Example 3: Determine $f^{1}(x)$ if $f(x) = x^{2}$ by using the first principle

$f(x) = x^{2}$
$f(x + h) = (x + h)^{2}$
$= x^{2} + 2xh + h^{2}$
$f(x + h) \quad - f(x) = 2xh + h^{2}$

$$f^{1}(x) = \lim_{h \Rightarrow 0} \frac{f(x+h) \qquad f(x)}{h}$$
$$= \lim_{h \Rightarrow 0} \frac{\cancel{h}(2x+h)}{\cancel{h}}$$
$$= \operatorname{Lim}_{h \Rightarrow 0} (2x + h)$$
$$= 2x.$$

Example 4: Pre-requisite knowledge
Easy way to expand $(x + h)^{3}$

a. **Write 1 +3 +3 +1**
b. **Write descending powers of x^{3} from LHS.**
c. **Write descending powers of h^{3} from RHS.**

$f(x) = x^{3}$
$f(x + h) = (x + h)^{3}$
$= x^{3} + 3x\,h^{2} + 3xh^{2} + h^{3}$
$f(x + h) \quad - f(x) = 3x\,h^{2} + 3xh^{2} + h^{3}$

$$f^{1}(x) = \lim_{h \Rightarrow 0} \frac{f(x+h) \qquad f(x)}{h}$$
$$= \lim_{H \Rightarrow 0} \frac{\cancel{h}(3x^{2} + 3xh + h^{2})}{\cancel{h}}$$
$$= \lim_{h \Rightarrow 0} (3x^{2} + 3xh + h^{2})$$
$$= 3x^{2}.$$

Example 5:

$f(x) = \frac{1}{x}$
$f(x + h) = \frac{1}{(x + h)}$
$f(x + h) \quad - F(x) = \frac{1}{(x+h)} \qquad \frac{1}{x}$
$= \frac{x \quad x \quad h}{x(x+h)}$

$$f^{1}(x) = \lim_{h \Rightarrow 0} \frac{f(x+h) \qquad f(x)}{h}$$
$$= \lim_{h \Rightarrow 0} \frac{\cancel{h}}{\cancel{h}x(x+h)}$$
$$= \operatorname{Lim}_{h \Rightarrow 0} \frac{1}{x^{2} + xh}$$
$$= \frac{1}{x^{2}}$$

Example 6:

6.1 **Given:** $g(x) = \frac{2x^{2} + x \ 1+}{2x - 1}$

6.1.1 **For which value of x is g(x) undefined?**
6.1.2 **The following table gives values of g(x) as x approaches 0,5. Give , correct to three decimal places, the missing value of g(0,501)**

x	**0**	**0.4**	**0.49**	**0.499**	**0.501**	**0.51**
g	**1**	**1.4**	**1.49**	**1.499**		**1.51**

6.1.3 **Find:** $\lim_{x \Rightarrow 0,5} g(x)$

Solutions:
6.1.1 $x = \frac{1}{2}$
6.1.2 **1.501**
6.1.3 **1.5**