MATHEMATICS

EXAM-BASED QUESTIONS

K12 SCHOOLS
YOUR EDUCATION PORTAL

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SAMPLE QUESTIONS: PAPER 1

QUESTION ONE

1.1 Solve for x , rounded off to TWO decimal places where necessary:

1.1.1 $x^2 + 2x = 3$ (3)
1.1.2 $x(3x+5) = -1$ (5)
1.1.3 $x^2 - 5x = 6$ (5)

1.2 Solve for x and y in the following simultaneous equations:

$3y + x = 2$

$y^2 + x = xy + y$ (7)

1.3 Calculate the value of:

$(1\ 000\ 000\ 000\ 000)^2 - (999\ 999\ 999\ 999)^2$

Show all calculations. (3)
[23]

QUESTION TWO

2.1 Determine how many terms the following sequence has:

-3 ; 2 ; 7 ; 12 ; ……; 242 (3)

2.2 Consider the sequence:

2 ; 8 ; 16 ; 26 ; ……

2.2.1 If the pattern continues consistently , write down the next two terms of the sequence. (2)
2.2.2 Calculate a formula for the nth term of the sequence. (4)
2.2.3 Will 12 369 be a term in this sequence? Show all calculations. (3)
[12]

QUESTION THREE

3.1 Given the geometric series:

$4(x-3) + 2(x-3)^2 + (x-3)^3 + \ldots$

3.1.1 Determine the nth term of the series in terms of x. (3)
3.1.2 For what values of x will the series converge? (3)
3.1.3 Calculate the sum to infinity if x = 4. (3)
[9]

QUESTION FOUR

4.1 Mabutho was given the following series to add.

1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10

He worked it out in the following way:

Sum to 1 term $= (T_1 \times T_2) \div 2 = (1 \times 2) \div 2 = 1$
Sum to 2 terms $= (T_2 \times T_3) \div 2 = (2 \times 3) \div 2 = 3$
Sum to 3 terms $= (T_3 \times T_4) \div 2 = (3 \times 4) \div 2 = 6$

Sum to 10 terms $= (10 \times 11) \div 2 = 55$

4.1.1 Is Mabutho correct ? Verify his answer using any other appropriate method. (3)

4.1.2 Hence, consider the following pattern:

$1^3 = 1^2 = 1$
$1^3 + 2^3 = (1+2)^2 = 9$
$1^3 + 2^3 + 3^3 = (1+2+3)^2 = 36$
$1^3 + 2^3 + 3^3 + 4^3 = (1+2+3+4)^2 = 100$

Without calculators , determine the value of:

$\sqrt{1^3 + 2^3 + 3^3 + 4^3 + \ldots\ldots + 97^3 + 98^3 + 99^3 + 100^3}$ (3)
[6]

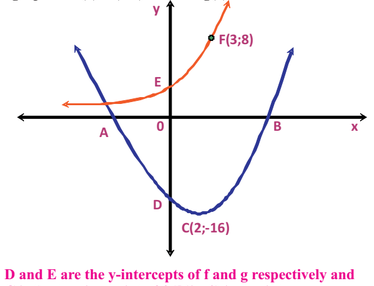
QUESTION FIVE

5.1 **Given** $g(x) = \frac{3}{x-2} + 1$

5.1.1 Write down the equations of the asymptotes of g. (2)
5.1.2 Calculate the intercepts of g with the axes. (2)
5.1.3 Draw a neat sketch of g , showing the asymptotes and the intercepts with the axes. (4)
5.1.4 The graph of g is reflected across the line y = -1 , and then shifted 2 units to the left. Write down the equation of the new graph. (2)
[10]

QUESTION SIX

6.1 The graphs of $f(x) = (x-2)^2 - 16$ and $g(x) = a^x$ are sketched below.

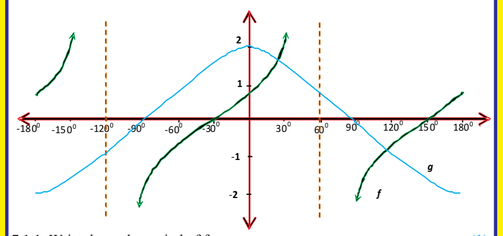


D and E are the y-intercepts of f and g respectively and C is the turning point of f. F(3 ; 8) is a point on g.

6.1.1 Determine the co-ordinates of A and B. (4)
6.1.2 Calculate the length of DE. (2)
6.1.3 Calculate the value of a. (2)
6.1.4 Use the graphs to determine the value/s of x for which $f(x).g(x) < 0$ (3)
6.1.5 Write down the equation of the resultant function when g(x) is reflected about the line x = 0. (2)
6.1.6 Write down the equation of $g^{-1}(x)$ in the form y = …….. (2)
6.1.7 Sketch the graph of $g^{-1}(x)$. (3)
[18]

QUESTION SEVEN

7.1 Sketched below are the graphs of the functions
$f(x) = \tan(x+30^0)$ and $g(x) = 2\cos x$ for $x \in [-180^0 ; 180^0]$



7.1.1 Write down the period of f. (1)
7.1.2 Write down the amplitude of h(x) = 3g(x) (2)
7.1.3 Give the new range of g if g undergoes a negative vertical shift of 1 unit. (2)
7.1.4 The period of g is doubled and the amplitude remains the same, write down two equations for the resultant functions. (2)
[7]

QUESTION EIGHT

8.1 Determine how long, in years , it will take R7860 to treble in value if it is invested at 8,2% p. a compounded half-yearly. (4)
8.2 Peter wants to buy a car costing R266 000. He pays a deposit of 20 % and finances the balance with a bank loan at a rate of 14,4 % per annum compounded monthly.
8.2.1 Calculate the loan amount that Peter needs to take from the bank. (2)
8.2.2 Calculate the monthly payment if he wishes to settle the loan in 5 yrs. (4)
8.2.3 After the 54th instalment , Peter decided to settle the loan with a lump sum payment. Calculate his outstanding balance immediately after the 54th instalment. (4)
[14]

QUESTION NINE

9.1 Determine $f'(x)$ from first principles , if $f(x) = -3x^2 + 4$ (5)

9.2 Determine using rules of differentiation:

9.2.1 $\frac{dy}{dx}$ if $y = \sqrt{x}\left(\frac{1}{\sqrt{x}}\right)^2$ (3)

9.2.2 $\frac{dy}{dx}$ if $y - xy = 1 - x^2$ (3)
[11]

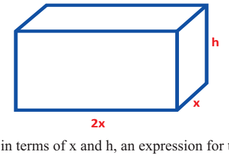
QUESTION TEN

10.1 Given $f(x) = 2x^3 - 3x^2 - 12x - 7$.

10.1.1 If f (-1) = 0 , calculate the x and y intercepts of f. (3)
10.1.2 Determine the co-ordinates of the turning points of the graph of f. (4)
10.1.3 Sketch the graph of f , showing all intercepts with the axes as well as the co-ordinates of the turning points. (4)
10.1.4 Determine the average rate of change of f from x = -1 to x =2. (2)
10.1.5 Determine the x-coordinate of the point of inflection of g(x) if g(x) = f(x-2) (2)
10.1.6 Determine the value/s of k for which $f(x) - k = 0$ will have three roots , of which two will be equal to each other. (2)
[17]

QUESTION ELEVEN

11. A rectangular container as shown in the sketch , is made of a thin metal alloy and without a lid, is of length 2x cm ,width x cm and height h cm.



11.1 Write down in terms of x and h, an expression for the total surface area of the container. (2)
11.2 If this area is 900 cm^2, show that $h = \frac{900}{6x} - \frac{2x^2}{6x}$ (2)
11.3 Show that the volume is given by: $V = 300x - \frac{2}{3}x^3$ (2)
11.4 Find the value of x for which **the volume V** is a maximum. (leave your answer in simplest surd form). (2)
[8]

QUESTION TWELVE

12. While preparing for the examinations , a learner created a weekly timetable for Mathematics and Physical Science. He planned it in such a way that the weekly roster must have periods for each of the two subjects. The following constraints were used in creating the timetable:

* there was to be at least one Mathematics and one Physical Science period.
* there had to be at least 7 periods altogether, but not more than 12.
* the number of Physical Science periods was to be at least twice the number of Mathematics periods.
* the timetable could not have more than 9 Physical Science periods.

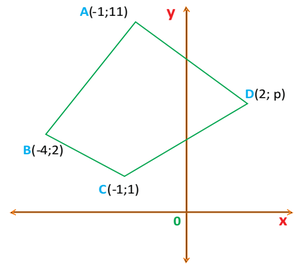
Let the number of Mathematics periods be x and the number of Physical Science periods be y.

12.1 Two of the constraints are x and y . Write down the other constraints as a system of inequalities. (4)
12.2 Use graph paper to sketch the inequalities, indicating clearly the feasible region. (5)
12.3 According to these constraints could the timetable have 5 Physical Science periods and 3 Mathematics periods? Give a reason for your answer. (2)
12.4 The object function T = 2x +3y must be maximized. Use a search line and your graph to determine how many periods of each subject must the timetable have in order to maximise T. (2)
12.5 In order to cater for the other subjects , the learner decides to minimize K = x + y subject to the above constraints. Determine the number of periods of Mathematics and Physical Science that minimizes K. (2)
[15] TOTAL : 150

SAMPLE QUESTIONS:PAPER 2

QUESTION ONE

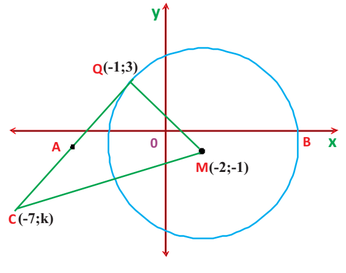
ABCD is a quadrilateral with vertices A(-1;11) , B(-4;2) , C(-1;1) and D(2;p).



1.1 Calculate the gradient of AB. (2)
1.2 Show that AB BC. (3)
1.3 Calculate the area of ? ABC. (3)
1.4 If it further given that the inclination of line AD is 135^0, show that p = 8. (3)
1.5 Determine the equation of AD. (3)
1.6 Calculate the size of BÂD correct to ONE decimal digit. (3)
1.7 Write down the co-ordinates of E , such that **AECD** becomes a parallelogram. (2)
[19]

QUESTION TWO

M(2;-1) is the centre of a circle in a Cartesian plane. Q(-1 ;3) is a point on the circle. The point T(-7 ; k) lies on the tangent to the circle at Q. A is the midpoint of the tangent CQ.



2.1 Calculate the length of the radius of the circle. (2)
2.2 Show that the equation of the circle is given by $x^2 + y^2 - 4x + 2y - 20 = 0$ (4)
2.3 Calculate the co-ordinates of B , the x intercept of the circle , correct to ONE decimal place. (3)
2.4 Calculate the gradient of QM. (2)
2.5 Hence ,determine the equation of the tangent CQ to the circle at Q in the form y = …….. (3)
2.6 Calculate the value of k. (2)
2.7 Determine the equation of the circle , centre at A , and passing through C and Q in the form $(x-a)^2 + (y-b)^2 = r^2$. (3)
[19]

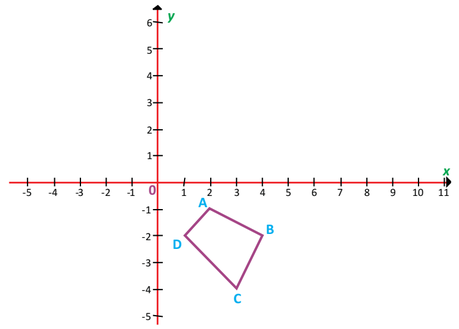
QUESTION THREE

3.1 The point $P(-2;\sqrt{5})$ lies in a Cartesian plane. Determine the co-ordinates of the image of P if:

3.1.1 P is reflected in the line y = x. (2)
3.1.2 P is rotated about the origin 180^0. (2)

3.2 Write down the general rule for a transformation T of ? PQR in the XOY plane described as follows:
- each point is rotated 90^0 in an anti- clockwise direction.
- each point is then enlarged by a factor 3 through the origin. (2)

3.3 The vertices of a polygon ABCD are shown. The co-ordinates are A(2;-1) , B(4;-2) , C(3;-4) and D(1;-2).



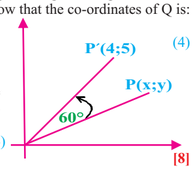
3.3.1 Each of the points of ABCD is rotated about the origin in an anti-clockwise direction through an angle of 90^0. Sketch and label the vertices of A B C D on Diagram 1in the DIAGRAM SHEET. (4)
3.3.2 If CD = $2\sqrt{2}$ units, write down the length of C D (2)
3.3.3 A B C D an enlargement of A B C D through the origin by a scale factor of 3. Write down the co-ordinates of D (2)
3.3.4 The area of ABCD is 4,5 square units. Calculate the area of A B C D (2)
3.3.5 Determine the ratio of $\frac{BD}{DB}$ (2)
[18]

QUESTION FOUR

4.1 The point C(x; y) is rotated about the origin through an angle of 300^0 in a clockwise direction to a new point Q. Show that the co-ordinates of Q is:

$\frac{1}{2}x \quad \frac{\sqrt{3}}{2}y \;;\; \frac{1}{2}y \quad \frac{\sqrt{3}}{2}x$ (4)

4.2 The diagram shows P and P (4;5).
The point P(x ; y) when rotated about the origin through an angle of 60° in an anti-clockwise direction becomes P (4;5). Determine the co-ordinates of P. (4)



[8]

QUESTION FIVE

5.1 If $\cos 17^0 = m$, write down the values of the following in terms of m:

5.1.1 $\cos 197^0$ (2) 5.1.2 $\sin 163^0$ (2)
5.1.3 Show that $\frac{\cos 17^0}{\tan 17^0} + \sin 17^0 = \frac{1}{\sqrt{1 - m^2}}$ (3)

5.2 In ? PQR , it is known that $\hat{P}$ is an obtuse angle.

It is further given that $5\sin\hat{Q} - 4 = 0$ and $\sin\hat{R} = \frac{5}{13}$

Calculate the value of sin P , without using a calculator. (6)

5.3 **Prove that:**

$$\frac{2\cos 105^0 . \cos 15^0}{\cos 30^0 \cos x - \sin 30^0 \sin x} = \frac{\sqrt{3}}{3}$$ (6)
[20]

QUESTION SIX

6.1 Given the identity: $\frac{2\sin^2 x}{1 - 2\sin x.\cos x} \quad \frac{\sin 2x}{\sin x - \cos x} \quad \frac{2\sin x}{\sin x - \cos x}$

6.1.1 Determine the value/s of x for which the identity is undefined if x [0°;180°] (3)
6.1.2 Prove the given identity. (5)

6.2 Given the expression: $\sin 2x + \cos 2x + \sin^2 x$
6.2.1 Show that: $\sin 2x + \cos 2x + \sin^2 x = \cos x(2\sin x + 1)$ (2)
6.2.2 Determine the general solution if $\sin 2x + \cos 2x + \sin^2 x = 0$. (4)
[14]

QUESTION SEVEN

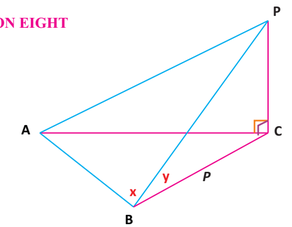
7.1 Sketch on the same set of axes the curves of:
$y = \cos \frac{1}{2}x$ and $y = \sin x + 1$ for $x \in [-180^0;180^0]$ (6)

7.2 Use your graphs to determine for which x:

7.2.1 $\frac{\sin x}{\cos\frac{x}{2}} \geq 1$ *if* $\sin x \neq 0$ (3)

7.2.2 $\cos \frac{1}{2}x . \sin x + \cos \frac{1}{2}x = 0$. (3)
[12]

QUESTION EIGHT



A, B and C are three points in the same horizontal plane. CP is a vertical tower. AC = BC = *p* units. The angle of elevation of P from B is y and PB̂A = x.

8.1 Give a reason why PA = PB. (1)
8.2 Express PA in terms of *p* and a trigonometric ratio of *y*. (2)
8.3 Show that BP̂A $= 180° - 2x$ (1)
8.4 Prove that $AB = \frac{2p.\cos x}{\cos y}$ (5)
8.5 If *p* = 100 metres ; y = 17,3° and x = 64,2° , calculate to the nearest metre:
8.5.1 AB (2)
8.5.2 the height of the tower CP. (2)
[13]

QUESTION NINE

The following data represents the scores of Mbuyeseni as a percentage in 12 assessment tasks.

42 54 41 62 73 56 69 71 55 38 48 62

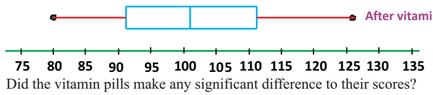
9.1 Calculate the mean percentage obtained by the learner in the 12 tasks. (2)
9.2 Calculate the standard deviation of the percentages obtained in the tasks. (2)
9.3 Determine the variance of the percentage scored in the assessments. (2)
9.4 Determine the number of tasks in which Mbuyeseni obtained a score that was within one standard deviation of the mean. (2)
[8]

QUESTION TEN

In an experiment 15 children were given an IQ test. The following are their results:

80 105 90 105 102 95 101 112 83 106 87 107 125 116 110

10.1 Determine the median IQ score of the learners. (1)
10.2 Calculate the lower and upper quartiles. (2)
10.3 Thereafter they were given **vitamins and retested** to check the effect of the vitamins on their performance. The box and whisker diagram below shows the 5 number summary of their results after taking the vitamins and then taking the test. Draw a box and whisker plot of the original scores before taking the vitamins in Diagram 2 in the DIAGRAM sheet. (3)



10.4 Did the vitamin pills make any significant difference to their scores? Comment on the similarities or differences between the spread of scores. (2)
[8]

Questions continued in next article: